PROXOP Report

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Astro 321

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I. INTRODUCTION

With the space flight, the ability to rendezvous orbiting satellites has become more important. Using several different techniques, I will analyze their differences in order to determine the best method of calculating proximity operations. In addition, the Clohessy-Wiltshire method and numerical methods were compared to determine their relative success at calculating proximity operations. Also compared were the Euler method and RK4 method to determine their shortcomings and the effect of the time step upon them. Both numerical and analytical methods make several assumptions in order to simplify the math technique. First, we are assuming two-body motion and can apply the two-body equation of motion. Also with both spacecraft in close proximity to each other, we can assume that the relative perturbation between them is zero. In addition, we have assumed that the target spacecraft is in a circular orbit.

II. MATH TECHNIQUE

In order to calculate the relative motion of proximity operations we need to create another reference frame. The XYZ reference frame is a non-inertial reference frame that is centered at the target spacecraft. The X direction follows radially along the position vector. The Y vector is in the direction of satellite motion and is in the plane of the satellite orbit. The Z direction follows the right hand rule and is perpendicular to the plane of satellite motion. With a known position vector for both the target and interceptor, we can calculate the relative range vector. Equation 1 shows this equation.

(1)

Applying the two-body equation of motion and binomial expansion, we can arrive at a state space representation of the relative motion of our interceptor in the XYZ reference frame. Equation 2 shows this state space representation.

(2)

We can apply this state space equation and arrive at the relative motion of the interceptor. To calculate the relative motion of the interceptor using numerical methods we first apply the position and velocity vectors. Applying the first derivative and the two-body equation of motion, we will arrive at Equation 3.

(3)

For the Euler method, we can apply a first order Taylor series to propagate the interceptor forward in time. This allows us to calculate the new position of the interceptor after a discrete time step. For the Euler method, Equation 4 shows us the initial values for for both the interceptor and target.

(4)

Equation 4 in conjunction with Equation 3 will yield the position vector of both the target and interceptor. Using Equation 1, we can then calculate the relative range of the interceptor to the target. In contrast, RK4 uses a different approach to propagate the interceptor forward. RK4 weights the derivatives in order to propagate the data. Equation 5-9 shows the equations used in RK4.

(5)

(6)

(7)

(8)

(9)

III. ANALYSIS

In order to compare the different proximity operation methods, I needed to test each method with several different initial conditions. The initial conditions were chosen to best test the methods. In addition, the choice of initial conditions must enable us to see the direct effect of each condition. Therefore, the initial conditions I chose only change one value at a time rather than many. This allows me to see the different effects between the methods. Using an initial x velocity of 1000 m/sec, we can see in Figure 1 the different results we obtain.

FIGURE 1. XY Plot initial x velocity

With this initial condition, we expect our interceptor to slow down as the craft moves into a higher slower orbit. As the craft moves through the orbit, their relative distance will decrease. This pattern continues as the interceptor is in an elliptical movement around each other. The numerical methods predict this kind of behavior to a different degree. Because both numerical methods are estimations, there exists lots of truncation error. This error causes a slight deviation that gradually increases as more error is continually compounded. A second initial condition that was tested involved an initial y position of 1000 meters. Figure 2 shows the different results obtained from this initial condition.

FIGURE 2. XY Plot – Initial 1000 meter y position

With an initial position of 1000 meters in the y direction, we would expect the spacecraft to stay at that distance for the entirety of the orbit. The spacecraft would lie 1000 meters in front of the target in the direction of the spacecraft velocity. The interceptor would simply lead the target. However, both numerical methods do not support this. The RK4 and Euler data shows that the interceptor would lie in an elliptical orbit with respect to the target. After approximately 80 minutes, the Euler data begins to diverge. Since we have only used a single order to approximate the Euler method there exists lots of truncation error. Using this data, I have determined that the analytical Clohessy-Wiltshire equations are the most accurate method to determine proximity operations. Next to, compare both numerical methods we can adjust the time step to determine its effect upon error. Figures 3 and 4 show the same initial 1000 x position with two different time steps.

FIGURE 3. XY Plot 1 sec time step

This data shows that both numerical methods approximate the same solution until approximately 70 minutes. At this point, both methods begin to diverge. Figure 4 shows the effect of a 60 sec time step upon the data.

FIGURE 4. XY Plot 60 sec time step

With a larger time step, both methods seem to fall apart. Neither method is able to calculate the relative position of the interceptor.

IV. RECOMMENDATIONS

The most accurate method remains the analytical Clohessy-Wiltshire equations. The choice of time step is irrelevant as the data is analytically derived and is not a function of previous data. The numerical methods can be improved by using more terms. This will decrease the error and create a more accurate model. With more terms, the data can be correct for a longer interval before the effect of compounded error sets in. In addition, we assumed that the relative perturbations between interceptor and target were irrelevant. However, in order to improve our approximation we should include this data into our calculations. This will create a model that better describes the relative position of the interceptor and target.

APPENDIX A – Data

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\* PROXOPS \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

Target Satellite Alt = 200.000 km

Interceptor relative position to target:

1000.0000 x 0.0000 y 0.0000 z m

Interceptor relative velocity to target:

0.0000 x 0.0000 y 0.0000 z m/sec

Step size for numerical integration = 1.00 sec

RK4 Solutions: Euler Approximation Solutions:

Time X Y Z X Y Z

(min) (km) (km)

0.0000 1.0000 0.0000 0.0000 1.0000 0.0000 0.0000

2.0000 1.0101 -0.1430 0.0000 1.0099 -0.1430 0.0000

4.0000 1.0400 -0.2916 0.0000 1.0397 -0.2917 0.0000

6.0000 1.0893 -0.4515 0.0000 1.0887 -0.4516 0.0000

8.0000 1.1569 -0.6281 0.0000 1.1560 -0.6282 0.0000

10.0000 1.2415 -0.8263 0.0000 1.2400 -0.8264 0.0000

12.0000 1.3413 -1.0508 0.0000 1.3392 -1.0508 0.0000

14.0000 1.4544 -1.3055 0.0000 1.4515 -1.3054 0.0000

16.0000 1.5784 -1.5940 0.0000 1.5745 -1.5935 0.0000

18.0000 1.7109 -1.9190 0.0000 1.7057 -1.9179 0.0000

20.0000 1.8492 -2.2826 0.0000 1.8425 -2.2805 0.0000

22.0000 1.9904 -2.6859 0.0000 1.9820 -2.6826 0.0000

24.0000 2.1318 -3.1295 0.0000 2.1214 -3.1243 0.0000

26.0000 2.2705 -3.6130 0.0000 2.2579 -3.6054 0.0000

28.0000 2.4037 -4.1352 0.0000 2.3886 -4.1246 0.0000

30.0000 2.5286 -4.6943 0.0000 2.5111 -4.6799 0.0000

32.0000 2.6428 -5.2875 0.0000 2.6227 -5.2684 0.0000

34.0000 2.7440 -5.9114 0.0000 2.7212 -5.8869 0.0000

36.0000 2.8301 -6.5621 0.0000 2.8048 -6.5313 0.0000

38.0000 2.8993 -7.2351 0.0000 2.8717 -7.1972 0.0000

40.0000 2.9503 -7.9254 0.0000 2.9205 -7.8795 0.0000

42.0000 2.9820 -8.6277 0.0000 2.9505 -8.5731 0.0000

44.0000 2.9938 -9.3364 0.0000 2.9610 -9.2725 0.0000

46.0000 2.9854 -10.0458 0.0000 2.9518 -9.9721 0.0000

48.0000 2.9569 -10.7502 0.0000 2.9231 -10.6665 0.0000

50.0000 2.9090 -11.4441 0.0000 2.8756 -11.3502 0.0000

52.0000 2.8426 -12.1221 0.0000 2.8101 -12.0181 0.0000

54.0000 2.7590 -12.7790 0.0000 2.7280 -12.6652 0.0000

56.0000 2.6598 -13.4102 0.0000 2.6309 -13.2871 0.0000

58.0000 2.5472 -14.0116 0.0000 2.5207 -13.8800 0.0000

60.0000 2.4232 -14.5797 0.0000 2.3994 -14.4403 0.0000

62.0000 2.2905 -15.1116 0.0000 2.2696 -14.9653 0.0000

64.0000 2.1516 -15.6052 0.0000 2.1336 -15.4531 0.0000

66.0000 2.0093 -16.0592 0.0000 1.9940 -15.9021 0.0000

68.0000 1.8666 -16.4728 0.0000 1.8535 -16.3119 0.0000

70.0000 1.7261 -16.8465 0.0000 1.7148 -16.6824 0.0000

72.0000 1.5909 -17.1813 0.0000 1.5805 -17.0147 0.0000

74.0000 1.4635 -17.4789 0.0000 1.4531 -17.3103 0.0000

76.0000 1.3465 -17.7420 0.0000 1.3350 -17.5714 0.0000

78.0000 1.2423 -17.9739 0.0000 1.2285 -17.8011 0.0000

80.0000 1.1529 -18.1785 0.0000 1.1356 -18.0029 0.0000

82.0000 1.0801 -18.3601 0.0000 1.0579 -18.1808 0.0000

84.0000 1.0253 -18.5238 0.0000 0.9969 -18.3393 0.0000

86.0000 0.9896 -18.6749 0.0000 0.9538 -18.4833 0.0000

88.0000 0.9738 -18.8187 0.0000 0.9292 -18.6180 0.0000

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Time X Y Z X Y Z

(min) (km) (km)

0.0000 0.0000 1.0000 0.0000 0.0000 1.0000 0.0000

2.0000 0.1415 0.9799 0.0000 0.1415 0.9800 0.0000

4.0000 0.2802 0.9199 0.0000 0.2801 0.9201 0.0000

6.0000 0.4133 0.8212 0.0000 0.4130 0.8215 0.0000

8.0000 0.5380 0.6859 0.0000 0.5376 0.6863 0.0000

10.0000 0.6519 0.5167 0.0000 0.6512 0.5173 0.0000

12.0000 0.7527 0.3169 0.0000 0.7516 0.3177 0.0000

14.0000 0.8383 0.0906 0.0000 0.8368 0.0917 0.0000

16.0000 0.9070 -0.1576 0.0000 0.9051 -0.1561 0.0000

18.0000 0.9575 -0.4228 0.0000 0.9549 -0.4208 0.0000

20.0000 0.9887 -0.6996 0.0000 0.9854 -0.6970 0.0000

22.0000 1.0000 -0.9825 0.0000 0.9960 -0.9791 0.0000

24.0000 0.9912 -1.2658 0.0000 0.9863 -1.2613 0.0000

26.0000 0.9624 -1.5437 0.0000 0.9567 -1.5381 0.0000

28.0000 0.9142 -1.8106 0.0000 0.9077 -1.8038 0.0000

30.0000 0.8477 -2.0613 0.0000 0.8403 -2.0530 0.0000

32.0000 0.7640 -2.2906 0.0000 0.7559 -2.2808 0.0000

34.0000 0.6650 -2.4939 0.0000 0.6562 -2.4827 0.0000

36.0000 0.5526 -2.6672 0.0000 0.5432 -2.6545 0.0000

38.0000 0.4290 -2.8069 0.0000 0.4192 -2.7929 0.0000

40.0000 0.2969 -2.9103 0.0000 0.2867 -2.8951 0.0000

42.0000 0.1587 -2.9752 0.0000 0.1483 -2.9592 0.0000

44.0000 0.0174 -3.0004 0.0000 0.0067 -2.9839 0.0000

46.0000 -0.1243 -2.9853 0.0000 -0.1353 -2.9686 0.0000

48.0000 -0.2635 -2.9303 0.0000 -0.2748 -2.9138 0.0000

50.0000 -0.3973 -2.8364 0.0000 -0.4091 -2.8205 0.0000

52.0000 -0.5232 -2.7056 0.0000 -0.5357 -2.6906 0.0000

54.0000 -0.6385 -2.5404 0.0000 -0.6520 -2.5266 0.0000

56.0000 -0.7410 -2.3443 0.0000 -0.7559 -2.3317 0.0000

58.0000 -0.8285 -2.1211 0.0000 -0.8452 -2.1098 0.0000

60.0000 -0.8994 -1.8754 0.0000 -0.9184 -1.8650 0.0000

62.0000 -0.9522 -1.6120 0.0000 -0.9740 -1.6023 0.0000

64.0000 -0.9858 -1.3364 0.0000 -1.0109 -1.3265 0.0000

66.0000 -0.9996 -1.0540 0.0000 -1.0285 -1.0431 0.0000

68.0000 -0.9932 -0.7705 0.0000 -1.0263 -0.7574 0.0000

70.0000 -0.9669 -0.4917 0.0000 -1.0046 -0.4750 0.0000

72.0000 -0.9211 -0.2231 0.0000 -0.9636 -0.2013 0.0000

74.0000 -0.8568 0.0298 0.0000 -0.9042 0.0585 0.0000

76.0000 -0.7752 0.2619 0.0000 -0.8276 0.2994 0.0000

78.0000 -0.6780 0.4686 0.0000 -0.7350 0.5167 0.0000

80.0000 -0.5672 0.6457 0.0000 -0.6285 0.7065 0.0000

82.0000 -0.4449 0.7897 0.0000 -0.5099 0.8649 0.0000

84.0000 -0.3137 0.8976 0.0000 -0.3815 0.9890 0.0000

86.0000 -0.1762 0.9673 0.0000 -0.2459 1.0763 0.0000

88.0000 -0.0351 0.9973 0.0000 -0.1055 1.1253 0.0000